The Parallelogram Method

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Abstract

The parallelogram method is a classic technique for converting historical earned premium to current rate levels. This paper demonstrates the parallelogram method.

1 Introduction

A common way to investigate the proper price of an insurance contract is to evaluate the historical performance of similar contracts. Part of this process may be to select an historical period and compute how much premium would have been earned over that period if the insurance in question were written at current rate levels. The parallelogram method is a popular way of estimating this on-level earned premium. See Werner (2009) for an introduction to pricing and an explanation of this method.

This paper demonstrates the basic parallelogram method. The implemented method yields on-level earned premium (or on-level earned premium factors) by period given historical rate change information.

Unlike the typical parallelogram method, the paper's implementation allows the rate at which premium is written to change over time, as specified by a step function. In theory this should result in more accurate on-level factors, although the extension of exposures method should perhaps be used instead if accuracy is important.

Most of the usual limitations of the parallelogram method still apply. For instance, this paper is not suitable if premium is not earned evenly over the policy term, if the mix of business has changed, if calendar details (e.g. that leap years are longer than common years) are significant, or if rate change information is not available.

2 Required Input Data

To apply the parallelogram method, you need a record of rate changes by year, where each rate change is expressed as a percent of premium before the rate change. This information

is put into a data frame as shown in figure 1.

Date (in years)	Rate Change (%)
2002.0	7.0
2003.5	-3.0
2004.1	12.0
2004.4	2.0
2005.5	10.0

Figure 1: Historical Rate Changes

The term length has also been set at 1.

The traditional parallelogram method assumes premium is written evenly. This implementation allows the written rate to vary; this can change the resulting on-level factors. Figure 2 shows the written premium rate as a step function. Each row indicates the start of an interval and the rate (in units of money per year) at which premium is written in that interval. Within each interval, premium is assumed to be written at a constant rate.

Period Start	Premium Rate
2001.0	30
2002.0	45
2003.0	75
2004.0	30
2005.0	55

Figure 2: Rate of Premium Written by Period

Finally, we need a list of non-overlapping periods that are interested in. The method will compute one on-level factor per period. They computed for each period as output. The periods chosen here are those between these years: (2001, 2002, 2003, 2004, 2005, 2006).

3 Method Output

3.1 Constant Written Rate

If the rate of written premium is not specified, a constant rate of 1 unit of premium per year is assumed. The resulting parallelogram is shown in figure 3. The numeric labels indicate

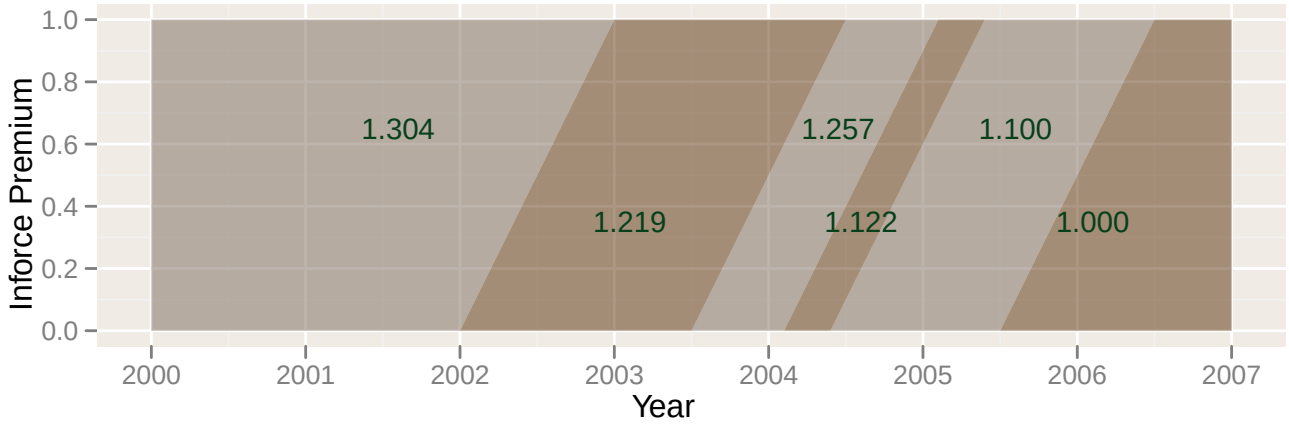

Figure 3: Basic Parallelogram

the rate level for each parallelogram-shaped region.

The final on-level factors by period are shown in figure 4.

Period Start	Period End	On-Level Factor
2001	2002	1.304
2002	2003	1.262
2003	2004	1.224
2004	2005	1.193
2005	2006	1.090

Figure 4: On-Level Factors

3.2 Variable Written Rate

If the rate at which premium is written changes, this should affect the on-level factors used. This implementation of the parallelogram method allows the written rate function to be specified as a step function. Using the assumptions of section 2, the inforce graph changes as shown in figure 5. In that plot, the dotted line represents specified the rate of premium written, while the numeric labels still represent the on-level factor for each rating period.

The on-level earned premium and earned premium factors that reflect the variable premium rate are shown in figure 6.

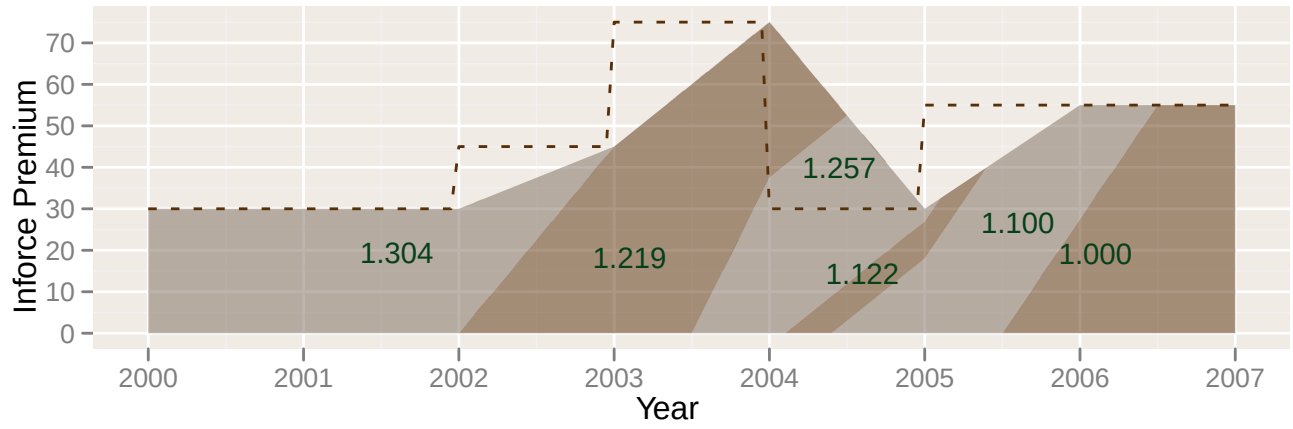

Figure 5: Parallelogram with Variable Premium Rate

Period Start	Period End	Earned Premium	On-Level Factor	On-Level Earned Premium
2001	2002	30	1.304	39.1
2002	2003	38	1.253	47.0
2003	2004	60	1.225	73.5
2004	2005	52	1.216	63.9
2005	2006	42	1.086	46.1

Figure 6: Variable Premium Results

Because the premium earned over any time period is just the area under the inforce premium curve divided by the term length, the traditional parallelogram method will work, although it is more complicated to calculate all the areas correctly. To obtain the variable premium results, the algorithm divides up all the regions into triangles and quadrilaterals and integrates them piece by piece.

For more details on this and on other ways the assumptions of parallelogram method can be relaxed, see Ross (1975).

4 Bibliography

1. Ross, James P. "Generalized Premium Formulae". *Proceedings of the Casualty Actuarial Society Casualty Actuarial Society*, Arlington, Virginia 1975: LXII 50-61. `http://www.casact.org/dare/index.cfm?fuseaction=view&abstrID=1673`

2. Werner, G. and Modlin, C. *Basic Ratemaking*. Casualty Actuarial Society, Version 2, October 2009. `http://www.casact.org/pubs/Werner_Modlin_Ratemaking.pdf`

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