FHSST Authors

The Free High School Science Texts: Textbooks for High School Students Studying the Sciences Physics Grades 10 - 12

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Chapter 6

Transverse Waves - Grade 10

6.1 Introduction

Waves occur frequently in nature. The most obvious examples are waves in water, on a dam, in the ocean, or in a bucket. We are most interested in the properties that waves have. All waves have the same properties, so if we study waves in water, then we can transfer our knowledge to predict how other examples of waves will behave.

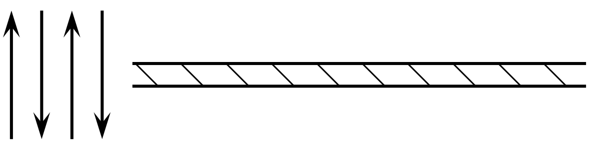
6.2 What is a transverse wave?

We have studied pulses in Chapter 5, and know that a pulse is a single disturbance that travels through a medium. A *wave* is a periodic, continuous disturbance that consists of a *train* of pulses.

Definition: Wave
A *wave* is a periodic, continuous disturbance that consists of a *train* of pulses.

Definition: Transverse wave
A *transverse wave* is a wave where the movement of the particles of the medium is perpendicular to the direction of propagation of the wave.

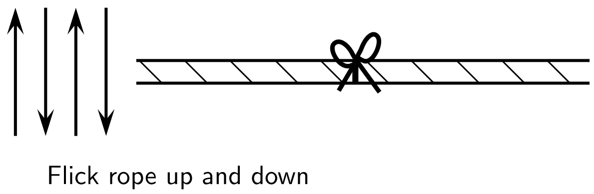
Activity :: Investigation : Transverse Waves
Take a rope or slinky spring. Have two people hold the rope or spring stretched out horizontally. Flick the one end of the rope up and down **continuously** to create a *train of pulses*.



Flick rope up and down

1. Describe what happens to the rope.
2. Draw a diagram of what the rope looks like while the pulses travel along it.

3. In which direction do the pulses travel?
4. Tie a ribbon to the middle of the rope. This indicates a particle in the rope.



5. Flick the rope continuously. Watch the ribbon carefully as the pulses travel through the rope. What happens to the ribbon?
6. Draw a picture to show the motion of the ribbon. Draw the ribbon as a dot and use arrows.

In the Activity, you have created waves. The medium through which these waves propagated was the rope, which is obviously made up of a very large number of particles (atoms). From the activity, you would have noticed that the wave travelled from left to right, but the particles (the ribbon) moved only up and down.

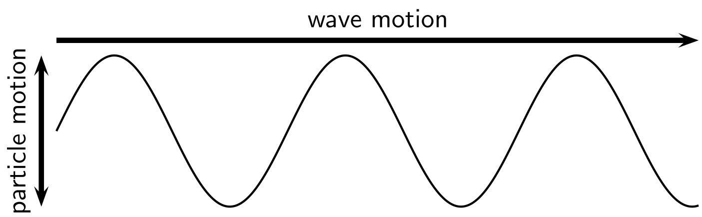


Figure 6.1: A transverse wave, showing the direction of motion of the wave perpendicular to the direction in which the particles move.

When the particles of a medium move at right angles to the direction of propagation of a wave, the wave is called *transverse*. For waves, there is no net displacement of the particles (they return to their equilibrium position), but there is a net displacement of the wave. There are thus two different motions: the motion of the particles of the medium and the motion of the wave.

6.2.1 Peaks and Troughs

Waves consist of moving *peaks* (or *crests*) and *troughs*. A peak is the highest point the medium rises to and a trough is the lowest point the medium sinks to.

Peaks and troughs on a transverse wave are shown in Figure 6.2.

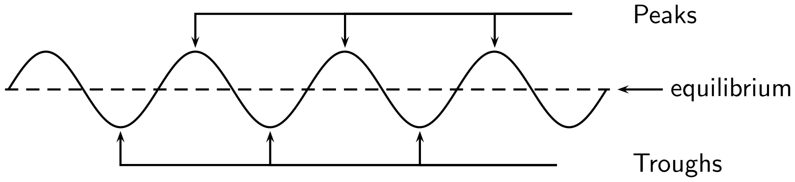


Figure 6.2: Peaks and troughs in a transverse wave.

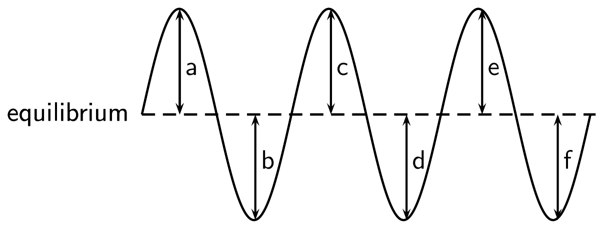
Definition: Peaks and troughs
A *peak* is a point on the wave where the displacement of the medium is at a maximum. A point on the wave is a *trough* if the displacement of the medium at that point is at a minimum.

6.2.2 Amplitude and Wavelength

There are a few properties that we saw with pulses that also apply to waves. These are amplitude and wavelength (we called this pulse length).

Definition: Amplitude
The *amplitude* is the maximum displacement of a particle from its equilibrium position.

Activity :: Investigation : Amplitude

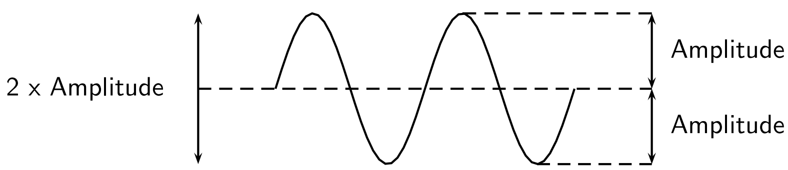


Fill in the table below by measuring the distance between the equilibrium and each peak and troughs in the wave above. Use your ruler to measure the distances.

Peak/Trough	Measurement (cm)
a	
b	
c	
d	
e	
f	

1. What can you say about your results?
2. Are the distances between the equilibrium position and each peak equal?
3. Are the distances between the equilibrium position and each trough equal?
4. Is the distance between the equilibrium position and peak equal to the distance between equilibrium and trough?

As we have seen in the activity on amplitude, the distance between the peak and the equilibrium position is equal to the distance between the trough and the equilibrium position. This distance is known as the *amplitude* of the wave, and is the characteristic height of wave, above or below the equilibrium position. Normally the symbol A is used to represent the amplitude of a wave. The SI unit of amplitude is the metre (m).



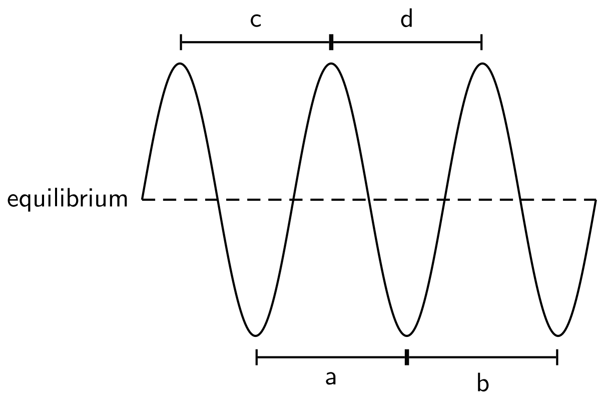
Worked Example 25: Amplitude of Sea Waves

Question: If the peak of a wave measures 2m above the still water mark in the harbour, what is the amplitude of the wave?

Answer

The definition of the amplitude is the height that the water rises to above when it is still. This is exactly what we were told, so the amplitude is 2m.

Activity :: Investigation : Wavelength



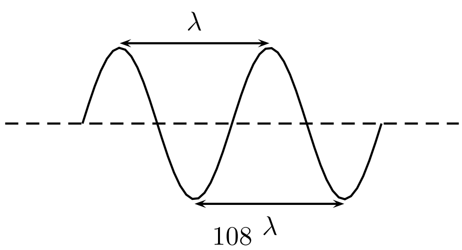
Fill in the table below by measuring the distance between peaks and troughs in the wave above.

	Distance(cm)
a	
b	
c	
d	

1. What can you say about your results?
2. Are the distances between peaks equal?
3. Are the distances between troughs equal?
4. Is the distance between peaks equal to the distance between troughs?

As we have seen in the activity on wavelength, the distance between two *adjacent* peaks is the same no matter which two adjacent peaks you choose. There is a fixed distance between the peaks. Similarly, we have seen that there is a fixed distance between the troughs, no matter which two troughs you look at. More importantly, the distance between two adjacent peaks is the same as the distance between two adjacent troughs. This distance is call the *wavelength* of the wave.

The symbol for the wavelength is λ (the Greek letter *lambda*) and wavelength is measured in metres (m).

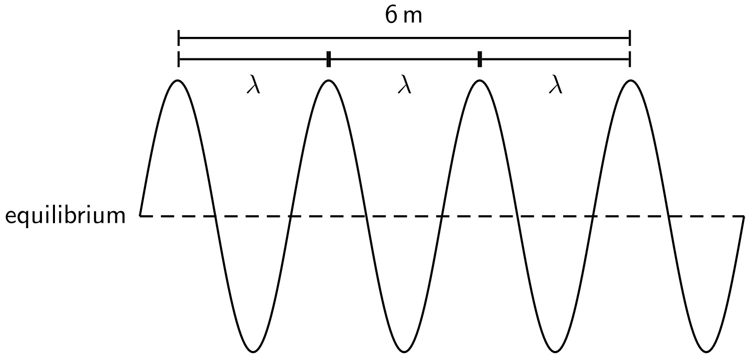


Worked Example 26: Wavelength

Question: The total distance between 4 consecutive peaks of a transverse wave is 6 m. What is the wavelength of the wave?

Answer

Step 1 : Draw a rough sketch of the situation



Step 2 : Determine how to approach the problem

From the sketch we see that 4 consecutive peaks is equivalent to 3 wavelengths.

Step 3 : Solve the problem

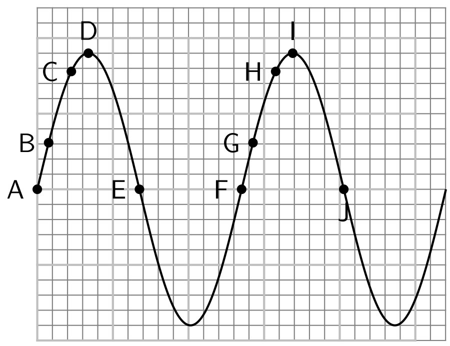
Therefore, the wavelength of the wave is:

$$\begin{aligned} 3\lambda &= 6\,\text{m} \\ \lambda &= \frac{6\,\text{m}}{3} \\ &= 2\,\text{m} \end{aligned}$$

6.2.3 Points in Phase

Activity :: Investigation : Points in Phase

Fill in the table by measuring the distance between the indicated points.



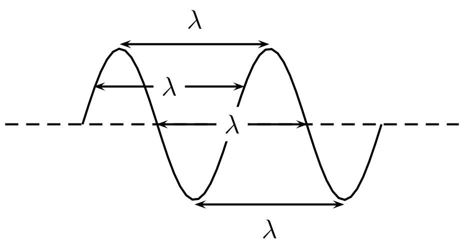
Points	Distance (cm)
A to F	
B to G	
C to H	
D to I	
E to J	

What do you find?

In the activity the distance between the indicated points was the same. These points are then said to be *in phase*. Two points in phase are separate by an integer (0,1,2,3,...) number of complete wave cycles. They do not have to be peaks or troughs, but they must be separated by a complete number of wavelengths.

We then have an alternate definition of the wavelength as the distance between any two adjacent points which are *in phase*.

Definition: Wavelength of wave
The wavelength of a wave is the distance between any two adjacent points that are in phase.



Points that are not in phase, those that are not separated by a complete number of wavelengths, are called *out of phase*. Examples of points like these would be A and C, or D and E, or B and H in the Activity.

6.2.4 Period and Frequency

Imagine you are sitting next to a pond and you watch the waves going past you. First one peak arrives, then a trough, and then another peak. Suppose you measure the time taken between one peak arriving and then the next. This time will be the same for any two successive peaks passing you. We call this time the *period*, and it is a characteristic of the wave.

The symbol T is used to represent the period. The period is measured in seconds (s).

Definition: The period (T) is the time taken for two successive peaks (or troughs) to pass a fixed point.

Imagine the pond again. Just as a peak passes you, you start your stopwatch and count each peak going past. After 1 second you stop the clock and stop counting. The number of peaks that you have counted in the 1 second is the *frequency* of the wave.

Definition: The frequency is the number of successive peaks (or troughs) passing a given point in 1 second.

The frequency and the period are related to each other. As the period is the time taken for 1 peak to pass, then the number of peaks passing the point in 1 second is $\frac{1}{T}$. But this is the frequency. So

$$f = \frac{1}{T}$$

or alternatively,

$$T = \frac{1}{f}.$$

For example, if a wave takes $\frac{1}{2}$ s to go by then the period of the wave is $\frac{1}{2}$ s. Therefore, the frequency of the wave is:

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{\frac{1}{2}\,\mathrm{s}} \\ &= 2\,\mathrm{s}^{-1} \end{aligned}$$

The unit of frequency is the Hertz (Hz) or s^{-1}.

Worked Example 27: Period and Frequency

Question: What is the period of a wave of frequency 10 Hz?
Answer
Step 1 : Determine what is given and what is required
We are required to calculate the period of a 10 Hz wave.
Step 2 : Determine how to approach the problem
We know that:

$$T = \frac{1}{f}$$

Step 3 : Solve the problem

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{1}{10\,\mathrm{Hz}} \\ &= 0{,}1\,\mathrm{m} \end{aligned}$$

Step 4 : Write the answer
The period of a 10 Hz wave is 0,1 m.

6.2.5 Speed of a Transverse Wave

In Chapter 3, we saw that speed was defined as

$$\text{speed} = \frac{\text{distance travelled}}{\text{time taken}}.$$

The distance between two successive peaks is 1 wavelength, λ. Thus in a time of 1 period, the wave will travel 1 wavelength in distance. Thus the speed of the wave, v, is:

$$v = \frac{\text{distance travelled}}{\text{time taken}} = \frac{\lambda}{T}.$$

However, $f = \frac{1}{T}$. Therefore, we can also write:

$$\begin{aligned} v &= \frac{\lambda}{T} \\ &= \lambda \cdot \frac{1}{T} \\ &= \lambda \cdot f \end{aligned}$$

We call this equation the *wave equation*. To summarise, we have that $v = \lambda \cdot f$ where

- v = speed in m·s^{-1}

- $\lambda =$ wavelength in m
- $f =$ frequency in Hz

Worked Example 28: Speed of a Transverse Wave 1

Question: When a particular string is vibrated at a frequency of 10 Hz, a transverse wave of wavelength 0,25 m is produced. Determine the speed of the wave as it travels along the string.

Answer

Step 1 : Determine what is given and what is required

- frequency of wave: $f = 10\,\text{Hz}$
- wavelength of wave: $\lambda = 0{,}25\,\text{m}$

We are required to calculate the speed of the wave as it travels along the string. All quantities are in SI units.

Step 2 : Determine how to approach the problem

We know that the speed of a wave is:

$$v = f \cdot \lambda$$

and we are given all the necessary quantities.

Step 3 : Substituting in the values

$$\begin{aligned} v &= f \cdot \lambda \\ &= (10\,\mathrm{Hz})(0{,}25\,\mathrm{m}) \\ &= 2{,}5\,\mathrm{m} \cdot \mathrm{s}^{-1} \end{aligned}$$

Step 4 : Write the final answer

The wave travels at 2,5 $\text{m}\cdot\text{s}^{-1}$ in the string.

Worked Example 29: Speed of a Transverse Wave 2

Question: A cork on the surface of a swimming pool bobs up and down once per second on some ripples. The ripples have a wavelength of 20 cm. If the cork is 2 m from the edge of the pool, how long does it take a ripple passing the cork to reach the shore?

Answer

Step 1 : Determine what is given and what is required

We are given:

- frequency of wave: $f = 1\,\text{Hz}$
- wavelength of wave: $\lambda = 20\,\text{cm}$
- distance of leaf from edge of pool: $d = 2\,\text{m}$

We are required to determine the time it takes for a ripple to travel between the cork and the edge of the pool.

The wavelength is not in SI units and should be converted.

Step 2 : Determine how to approach the problem

The time taken for the ripple to reach the edge of the pool is obtained from:

$$t = \frac{d}{v} \qquad \text{(from } v = \frac{d}{t}\text{)}$$

We know that

$$v = f \cdot \lambda$$

Therefore,

$$t = \frac{d}{f \cdot \lambda}$$

Step 3 : Convert wavelength to SI units

$$20\,\text{cm} = 0{,}2\,\text{m}$$

Step 4 : Solve the problem

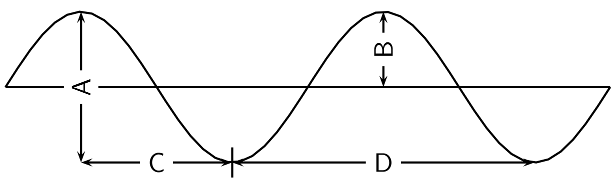
$$\begin{aligned} t &= \frac{d}{f \cdot \lambda} \\ &= \frac{2\,\text{m}}{(1\ \text{Hz})(0{,}2\,\text{m})} \\ &= 10\,\text{s} \end{aligned}$$

Step 5 : Write the final answer
A ripple passing the leaf will take 10 s to reach the edge of the pool.

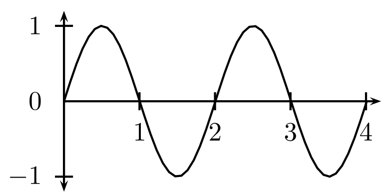
?

Exercise: Waves

1. List one property that distinguishes waves from matter.
2. When the particles of a medium move perpendicular to the direction of the wave motion, the wave is called a .... wave.
3. A transverse wave is moving downwards. In what direction do the particles in the medium move?
4. Consider the diagram below and answer the questions that follow:



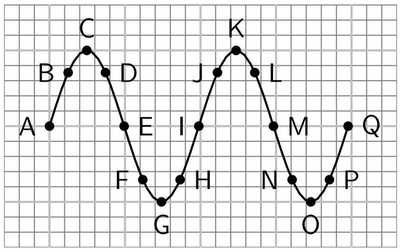
   (a) the wavelength of the wave is shown by letter ....
   (b) the amplitude of the wave is shown by letter ....
5. Draw 2 wavelengths of the following transverse waves on the same graph paper. Label all important values.
   (a) Wave 1: Amplitude = 1 cm, wavelength = 3 cm
   (b) Wave 2: Peak to trough distance (vertical) = 3 cm, peak to peak distance (horizontal) = 5 cm
6. You are given the transverse wave below.



   Draw the following:
   (a) A wave with twice the amplitude of the given wave.
   (b) A wave with half the amplitude of the given wave.

(c) A wave with twice the frequency of the given wave.
(d) A wave with half the frequency of the given wave.
(e) A wave with twice the wavelength of the given wave.
(f) A wave with half the wavelength of the given wave.
(g) A wave with twice the period of the given wave.
(h) A wave with half the period of the given wave.

7. A transverse wave with an amplitude of 5 cm has a frequency of 15 Hz. The horizontal distance from a crest to the nearest trough is measured to be 2,5 cm. Find the
   (a) period of the wave.
   (b) speed of the wave.
8. A fly flaps its wings back and forth 200 times each second. Calculate the period of a wing flap.
9. As the period of a wave increases, the frequency **increases/decreases/does not change**.
10. Calculate the frequency of rotation of the second hand on a clock.
11. Microwave ovens produce radiation with a frequency of 2 450 MHz (1 MHz = 10^6 Hz) and a wavelength of 0,122 m. What is the wave speed of the radiation?
12. Study the following diagram and answer the questions:

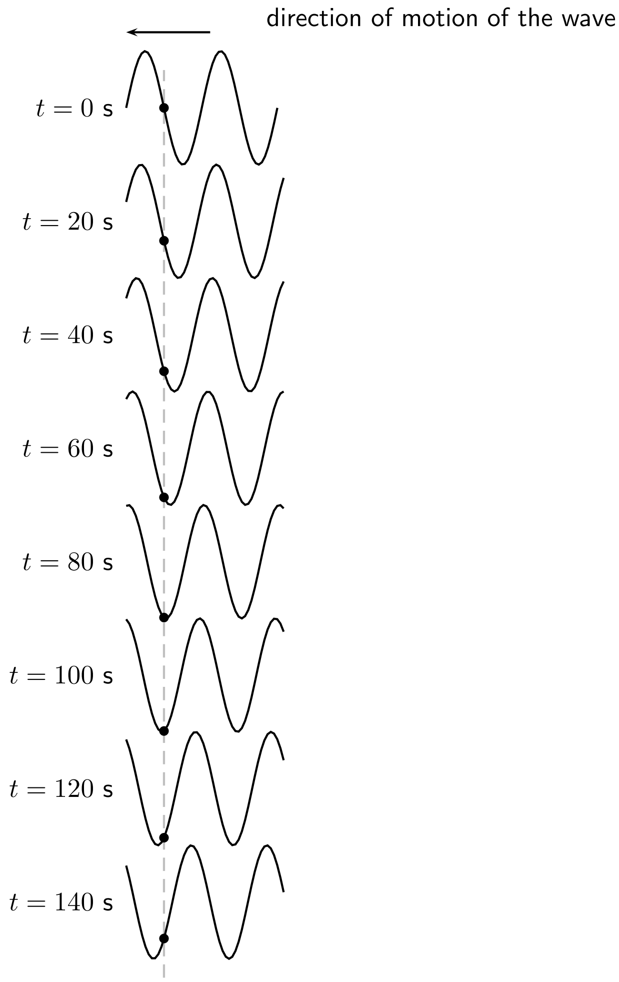


   (a) Identify two sets of points that are in phase.
   (b) Identify two sets of points that are out of phase.
   (c) Identify any two points that would indicate a wavelength.
13. Tom is fishing from a pier and notices that four wave crests pass by in 8 s and estimates the distance between two successive crests is 4 m. The timing starts with the first crest and ends with the fourth. Calculate the speed of the wave.

6.3 Graphs of Particle Motion

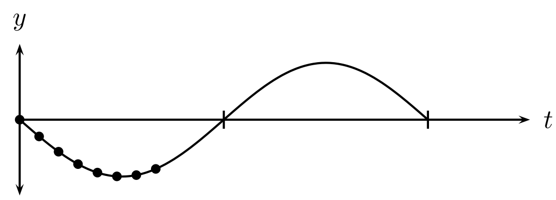
In Chapter 5, we saw that when a pulse moves through a medium, there are two different motions: the motion of the particles of the medium and the motion of the pulse. These two motions are at right angles to each other when the pulse is transverse. Since a transverse wave is a series of transverse pulses, the particle in the medium and the wave move in exactly the same way as for the pulse.

When a transverse wave moves through the medium, the particles in the medium **only** move up and down. We can see this in the figure below, which shows the motion of a single particle as a transverse wave moves through the medium.



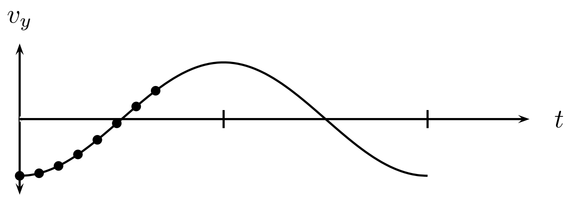
Important: A particle in the medium **only** moves up and down when a transverse wave moves through the medium.

As in Chapter 3, we can draw a graph of the particles' position as a function of time. For the wave shown in the above figure, we can draw the graph shown below.



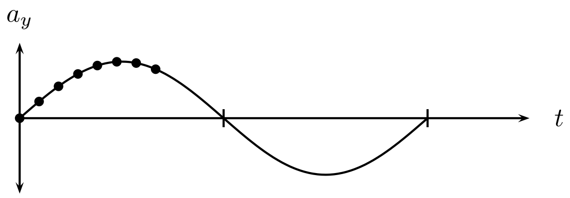
Graph of particle position as a function of time.

The graph of the particle's velocity as a function of time is obtained by taking the gradient of the position vs. time graph. The graph of velocity vs. time for the position vs. time graph above, is shown in the graph below.



Graph of particle velocity as a function of time.

The graph of the particle's acceleration as a function of time is obtained by taking the gradient of the velocity vs. time graph. The graph of acceleration vs. time for the position vs. time graph shown above, is shown below.

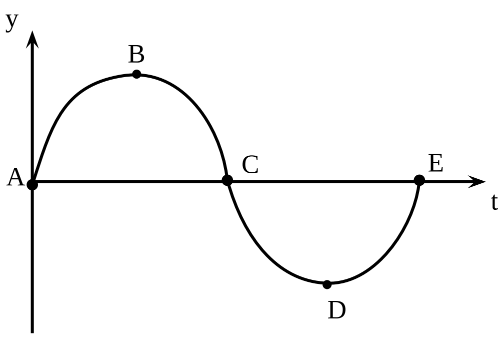


Graph of particle acceleration as a function of time.

As for motion in one dimension, these graphs can be used to describe the motion of the particle. This is illustrated in the worked examples below.

Worked Example 30: Graphs of particle motion 1

Question: The following graph shows the position of a particle of a wave as a function of time.



1. Draw the corresponding velocity vs. time graph for the particle.
2. Draw the corresponding acceleration vs. time graph for the particle.

Answer

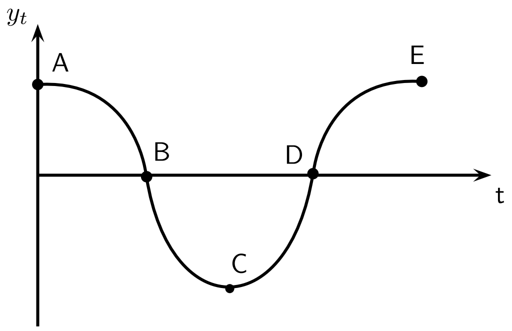
Step 1 : Determine what is given and what is required.

The y vs. t graph is given.
The v_y vs. t and a_y vs. t graphs are required.

Step 2 : Draw the velocity vs. time graph

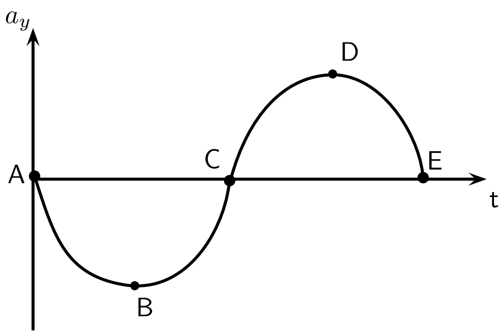
To find the velocity of the particle we need to find the gradient of the y vs. t graph at each time.

At point A the gradient is a maximum and positive.
At point B the gradient is zero.
At point C the gradient is a maximum, but negative.
At point D the gradient is zero.
At point E the gradient is a maximum and positive again.



Step 3 : Draw the acceleration vs. time graph

To find the acceleration of the particle we need to find the gradient of the v_y vs. t graph at each time.
At point A the gradient is zero.
At point B the gradient is negative and a maximum.
At point C the gradient is zero.
At point D the gradient is positive and a maximum.
At point E the gradient is zero.



Extension: Mathematical Description of Waves

If you look carefully at the pictures of waves you will notice that they look very much like *sine* or *cosine* functions. This is correct. Waves can be described by trigonometric functions that are functions of time or of position. Depending on which case we are dealing with the function will be a function of t or x. For example, a function of position would be:

$$y(x) = A\sin(k\frac{x}{\lambda})$$

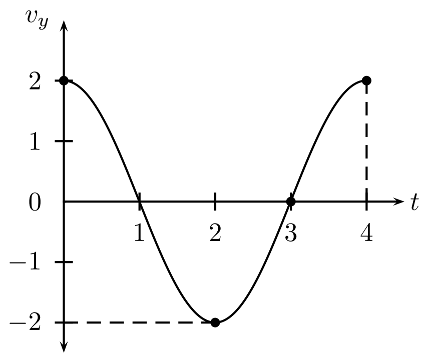
while a function of time would be:

$$y(t) = A\sin(k\frac{t}{T})$$

Descriptions of the wave incorporate the amplitude, wavelength, frequency or period and a phase shift.

Exercise: Graphs of Particle Motion

1. The following velocity vs. time graph for a particle in a wave is given.



   (a) Draw the corresponding position vs. time graph for the particle.
   (b) Draw the corresponding acceleration vs. time graph for the particle.

6.4 Standing Waves and Boundary Conditions

6.4.1 Reflection of a Transverse Wave from a Fixed End

We have seen that when a pulse meets a fixed endpoint, the pulse is reflected, but it is inverted. Since a transverse wave is a series of pulses, a transverse wave meeting a fixed endpoint is also reflected and the reflected wave is inverted. That means that the peaks and troughs are swapped around.

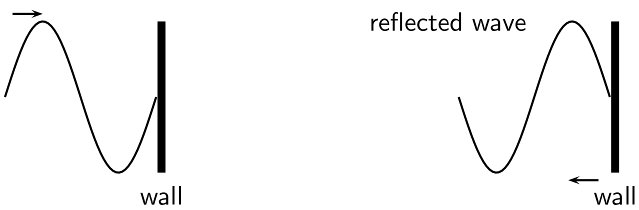


Figure 6.3: Reflection of a transverse wave from a fixed end.

6.4.2 Reflection of a Transverse Wave from a Free End

If transverse waves are reflected from an end, which is free to move, the waves sent down the string are reflected but do not suffer a phase shift as shown in Figure 6.4.

6.4.3 Standing Waves

What happens when a reflected transverse wave meets an incident transverse wave? When two waves move in opposite directions, through each other, interference takes place. If the two waves have the same frequency and wavelength then *standing waves* are generated.

Standing waves are so-called because they appear to be standing still.

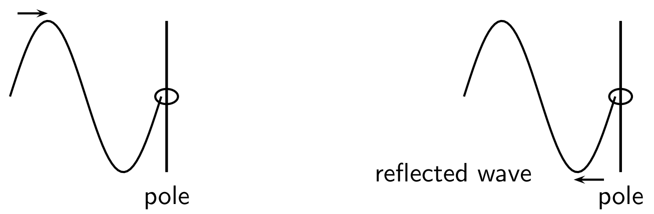


Figure 6.4: Reflection of a transverse wave from a free end.

Activity :: Investigation : Creating Standing Waves

Tie a rope to a fixed object such that the tied end does not move. Continuously move the free end up and down to generate firstly transverse waves and later standing waves.

We can now look closely how standing waves are formed. Figure 6.5 shows a reflected wave meeting an incident wave.

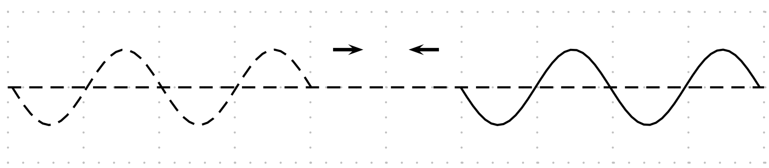


Figure 6.5: A reflected wave (solid line) approaches the incident wave (dashed line).

When they touch, both waves have an amplitude of zero:

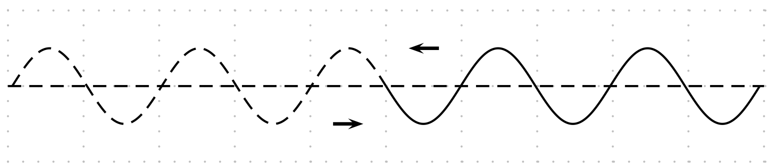


Figure 6.6: A reflected wave (solid line) meets the incident wave (dashed line).

If we wait for a short time the ends of the two waves move past each other and the waves overlap. To find the resultant wave, we add the two together.

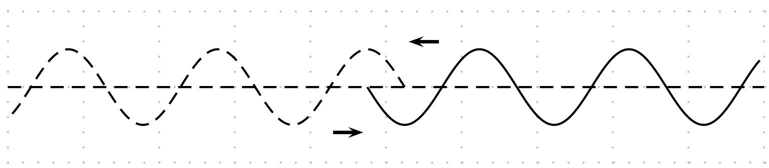
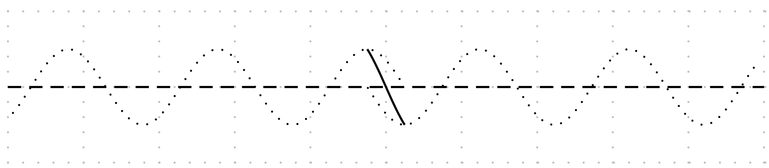
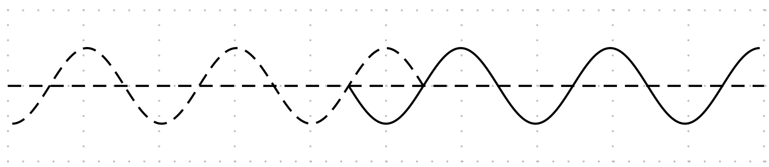


Figure 6.7: A reflected wave (solid line) overlaps slightly with the incident wave (dashed line).

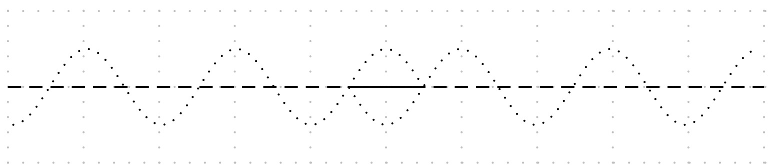
In this picture, we show the two waves as dotted lines and the sum of the two in the overlap region is shown as a solid line:



The important thing to note in this case is that there are some points where the two waves always destructively interfere to zero. If we let the two waves move a little further we get the picture below:

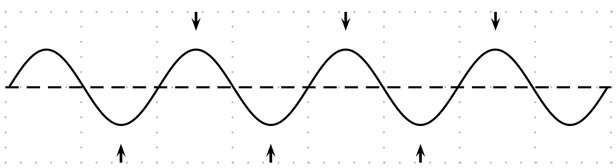


Again we have to add the two waves together in the overlap region to see what the sum of the waves looks like.

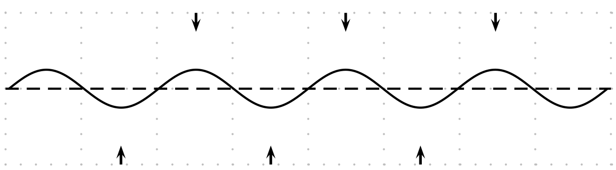


In this case the two waves have moved half a cycle past each other but because they are out of phase they cancel out completely.

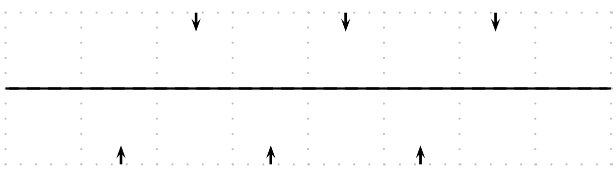
When the waves have moved past each other so that they are overlapping for a large region the situation looks like a wave oscillating in place. The following sequence of diagrams show what the resulting wave will look like. To make it clearer, the arrows at the top of the picture show peaks where maximum positive constructive interference is taking place. The arrows at the bottom of the picture show places where maximum negative interference is taking place.



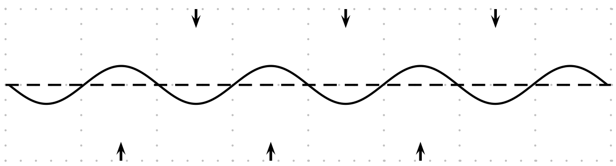
As time goes by the peaks become smaller and the troughs become shallower but they do not move.



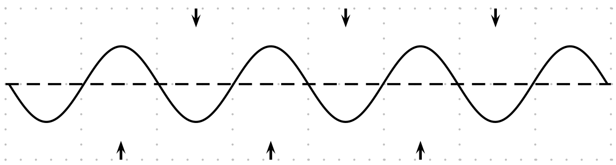
For an instant the entire region will look completely flat.



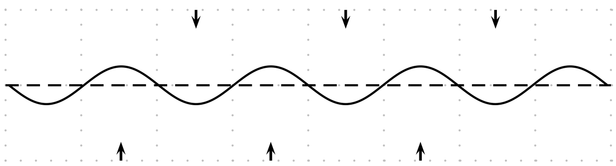
The various points continue their motion in the same manner.



Eventually the picture looks like the complete reflection through the x-axis of what we started with:



Then all the points begin to move back. Each point on the line is oscillating up and down with a different amplitude.



If we look at the overall result, we get a standing wave.

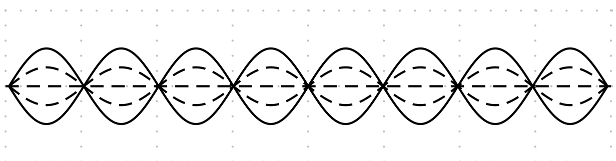
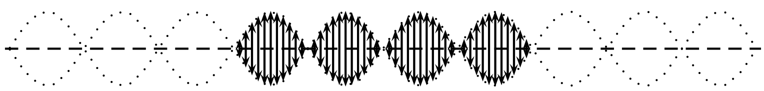


Figure 6.8: A standing wave

If we superimpose the two cases where the peaks were at a maximum and the case where the same waves were at a minimum we can see the lines that the points oscillate between. We call this the *envelope* of the standing wave as it contains all the oscillations of the individual points. To make the concept of the envelope clearer let us draw arrows describing the motion of points along the line.



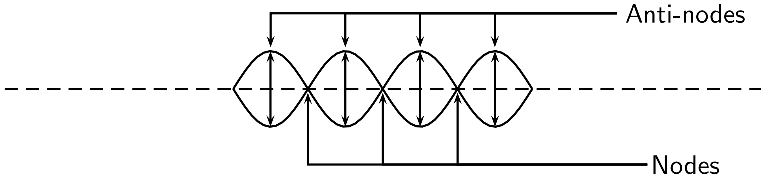
Every point in the medium containing a standing wave oscillates up and down and the amplitude of the oscillations depends on the location of the point. It is convenient to draw the envelope for the oscillations to describe the motion. We cannot draw the up and down arrows for every single point!

Interesting Fact

Standing waves can be a problem in for example indoor concerts where the dimensions of the concert venue coincide with particular wavelengths. Standing waves can appear as 'feedback', which would occur if the standing wave was picked up by the microphones on stage and amplified.

6.4.4 Nodes and anti-nodes

A node is a point on a wave where no displacement takes place. In a standing wave, a node is a place where the two waves cancel out completely as two waves destructively interfere in the same place. A fixed end of a rope is a node. An anti-node is a point on a wave where maximum displacement takes place. In a standing wave, an anti-node is a place where the two waves constructively interfere. A free end of a rope is an anti-node.



Definition: Node
A node is a point on a wave where no displacement takes place. In a standing wave, a node is a place where the two waves cancel out completely as two waves destructively interfere in the same place. A fixed end of a rope is a node.

Definition: Anti-Node
An anti-node is a point on a wave where maximum displacement takes place. In a standing wave, an anti-node is a place where the two waves constructively interfere. A free end of a rope is an anti-node.

Important: The distance between two anti-nodes is only $\frac{1}{2}\lambda$ because it is the distance from a peak to a trough in one of the waves forming the standing wave. It is the same as the distance between two adjacent nodes. This will be important when we work out the allowed wavelengths in tubes later. We can take this further because half-way between any two anti-nodes is a node. Then the distance from the node to the anti-node is half the distance between two anti-nodes. This is half of half a wavelength which is one quarter of a wavelength, $\frac{1}{4}\lambda$.

6.4.5 Wavelengths of Standing Waves with Fixed and Free Ends

There are many applications which make use of the properties of waves and the use of fixed and free ends. Most musical instruments rely on the basic picture that we have presented to create specific sounds, either through standing pressure waves or standing vibratory waves in strings.

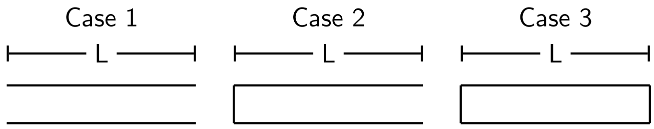
The key is to understand that a standing wave must be created in the medium that is oscillating. There are restrictions as to what wavelengths can form standing waves in a medium.

For example, if we consider a rope that can move in a pipe such that it can have

- both ends free to move (Case 1)
- one end free and one end fixed (Case 2)
- both ends fixed (Case 3).

Each of these cases is slightly different because the free or fixed end determines whether a node or anti-node will form when a standing wave is created in the rope. These are the main restrictions when we determine the wavelengths of potential standing waves. These restrictions are known as *boundary conditions* and **must** be met.

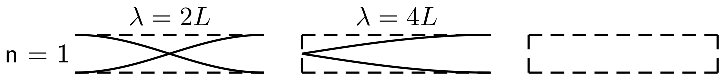
In the diagram below you can see the three different cases. It is possible to create standing waves with different frequencies and wavelengths as long as the end criteria are met.



The longer the wavelength the less the number of anti-nodes in the standing waves. We cannot have a standing wave with no anti-nodes because then there would be no oscillations. We use n to number the anti-nodes. If all of the tubes have a length L and we know the end constraints we can find the wavelength, λ, for a specific number of anti-nodes.

One Node

Let's work out the longest wavelength we can have in each tube, i.e. the case for $n = 1$.



Case 1: In the first tube, both ends must be nodes, so we can place one anti-node in the middle of the tube. We know the distance from one node to another is $\frac{1}{2}\lambda$ and we also know this distance is L. So we can equate the two and solve for the wavelength:

$$\begin{aligned} \frac{1}{2}\lambda &= L \\ \lambda &= 2L \end{aligned}$$

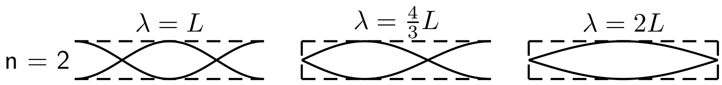
Case 2: In the second tube, one end must be a node and the other must be an anti-node. We are looking at the case with one anti-node we are forced to have it at the end. We know the distance from one node to another is $\frac{1}{2}\lambda$ but we only have half this distance contained in the tube. So :

$$\begin{aligned} \frac{1}{2}(\frac{1}{2}\lambda) &= L \\ \lambda &= 4L \end{aligned}$$

Case 3: Here both ends are closed and so we must have two nodes so it is impossible to construct a case with only one node.

Two Nodes

Next we determine which wavelengths could be formed if we had two nodes. Remember that we are dividing the tube up into smaller and smaller segments by having more nodes so we expect the wavelengths to get shorter.



Case 1: Both ends are open and so they must be anti-nodes. We can have two nodes inside the tube only if we have one anti-node contained inside the tube and one on each end. This means we have 3 anti-nodes in the tube. The distance between any two anti-nodes is half a wavelength. This means there is half wavelength between the left side and the middle and another half wavelength between the middle and the right side so there must be one wavelength inside the tube. The safest thing to do is work out how many half wavelengths there are and equate this to the length of the tube L and then solve for λ.

$$\begin{aligned} 2(\frac{1}{2}\lambda) &= L \\ \lambda &= L \end{aligned}$$

Case 2: We want to have two nodes inside the tube. The left end must be a node and the right end must be an anti-node. We can have one node inside the tube as drawn above. Again we can count the number of distances between adjacent nodes or anti-nodes. If we start from the left end we have one half wavelength between the end and the node inside the tube. The distance from the node inside the tube to the right end which is an anti-node is half of the distance to another node. So it is half of half a wavelength. Together these add up to the length of the tube:

$$\begin{aligned} \frac{1}{2}\lambda + \frac{1}{2}(\frac{1}{2}\lambda) &= L \\ \frac{2}{4}\lambda + \frac{1}{4}\lambda &= L \\ \frac{3}{4}\lambda &= L \\ \lambda &= \frac{4}{3}L \end{aligned}$$

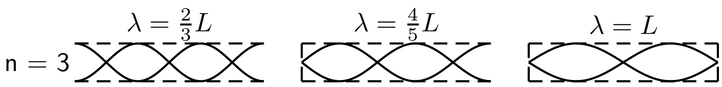
Case 3: In this case both ends have to be nodes. This means that the length of the tube is one half wavelength: So we can equate the two and solve for the wavelength:

$$\begin{aligned} \frac{1}{2}\lambda &= L \\ \lambda &= 2L \end{aligned}$$

Important: If you ever calculate a longer wavelength for more nodes you have made a mistake. Remember to check if your answers make sense!

Three Nodes

To see the complete pattern for all cases we need to check what the next step for case 3 is when we have an additional node. Below is the diagram for the case where $n = 3$.



Case 1: Both ends are open and so they must be anti-nodes. We can have three nodes inside the tube only if we have two anti-nodes contained inside the tube and one on each end. This means we have 4 anti-nodes in the tube. The distance between any two anti-nodes is half a wavelength. This means there is half wavelength between every adjacent pair of anti-nodes. We count how many gaps there are between adjacent anti-nodes to determine how many half wavelengths there are and equate this to the length of the tube L and then solve for λ.

$$\begin{aligned} 3(\frac{1}{2}\lambda) &= L \\ \lambda &= \frac{2}{3}L \end{aligned}$$

Case 2: We want to have three nodes inside the tube. The left end must be a node and the right end must be an anti-node, so there will be two nodes between the ends of the tube. Again we can count the number of distances between adjacent nodes or anti-nodes, together these add up to the length of the tube. Remember that the distance between the node and an adjacent anti-node is only half the distance between adjacent nodes. So starting from the left end we count 3 nodes, so 2 half wavelength intervals and then only a node to anti-node distance:

$$\begin{aligned} 2(\frac{1}{2}\lambda) + \frac{1}{2}(\frac{1}{2}\lambda) &= L \\ \lambda + \frac{1}{4}\lambda &= L \\ \frac{5}{4}\lambda &= L \\ \lambda &= \frac{4}{5}L \end{aligned}$$

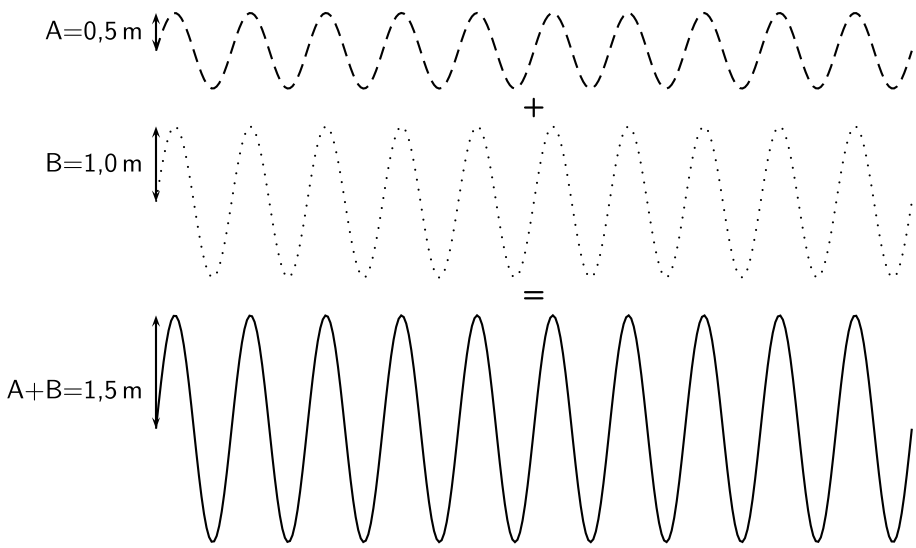
Case 3: In this case both ends have to be nodes. With one node in between there are two sets of adjacent nodes. This means that the length of the tube consists of two half wavelength sections:

$$\begin{aligned} 2(\frac{1}{2}\lambda) &= L \\ \lambda &= L \end{aligned}$$

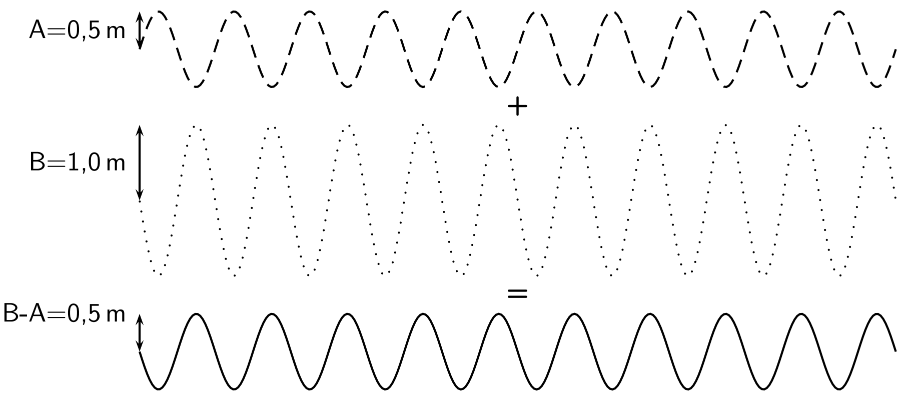
6.4.6 Superposition and Interference

If two waves meet interesting things can happen. Waves are basically collective motion of particles. So when two waves meet they both try to impose their collective motion on the particles. This can have quite different results.

If two identical (same wavelength, amplitude and frequency) waves are both trying to form a peak then they are able to achieve the sum of their efforts. The resulting motion will be a peak which has a height which is the sum of the heights of the two waves. If two waves are both trying to form a trough in the same place then a deeper trough is formed, the depth of which is the sum of the depths of the two waves. Now in this case, the two waves have been trying to do the same thing, and so add together constructively. This is called *constructive interference*.

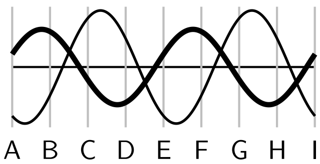


If one wave is trying to form a peak and the other is trying to form a trough, then they are competing to do different things. In this case, they can cancel out. The amplitude of the resulting wave will depend on the amplitudes of the two waves that are interfering. If the depth of the trough is the same as the height of the peak nothing will happen. If the height of the peak is bigger than the depth of the trough, a smaller peak will appear. And if the trough is deeper then a less deep trough will appear. This is *destructive interference*.



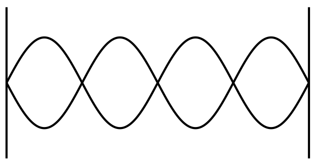
Exercise: Superposition and Interference

1. For each labelled point, indicate whether constructive or destructive interference takes place at that point.



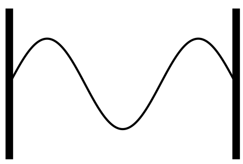
Position	Constructive/Destructive
A	
B	
C	
D	
E	
F	
G	
H	
I	

2. A ride at the local amusement park is called "Standing on Waves". Which position (a node or an antinode) on the ride would give the greatest thrill?
3. How many nodes and how many anti-nodes appear in the standing wave below?



4. For a standing wave on a string, you are given three statements:
   A you can have any λ and any f as long as the relationship, $v = \lambda \cdot f$ is satisfied.
   B only certain wavelengths and frequencies are allowed
   C the wave velocity is only dependent on the medium

   Which of the statements are true:
   (a) A and C only
   (b) B and C only
   (c) A, B, and C
   (d) none of the above
5. Consider the diagram below of a standing wave on a string 9 m long that is tied at both ends. The wave velocity in the string is 16 m·s^{-1}. What is the wavelength?

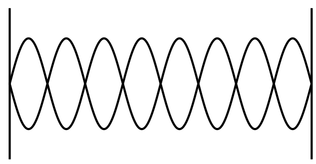


6.5 Summary

1. A wave is formed when a continuous number of pulses are transmitted through a medium.
2. A peak is the highest point a particle in the medium rises to.
3. A trough is the lowest point a particle in the medium sinks to.
4. In a transverse wave, the particles move perpendicular to the motion of the wave.
5. The amplitude is the maximum distance from equilibrium position to a peak (or trough), or the maximum displacement of a particle in a wave from its position of rest.
6. The wavelength (λ) is the distance between any two adjacent points on a wave that are in phase. It is measured in metres.
7. The period (T) of a wave is the time it takes a wavelength to pass a fixed point. It is measured in seconds (s).
8. The frequency (f) of a wave is how many waves pass a point in a second. It is measured in hertz (Hz) or s^{-1}.
9. Frequency: $f = \frac{1}{T}$
10. Period: $T = \frac{1}{f}$
11. Speed: $v = f\lambda$ or $v = \frac{\lambda}{T}$.
12. When a wave is reflected from a fixed end, the resulting wave will move back through the medium, but will be inverted. When a wave is reflected from a free end, the waves are reflected, but not inverted.
13. Standing waves.

6.6 Exercises

1. A standing wave is formed when:
   (a) a wave refracts due to changes in the properties of the medium
   (b) a wave reflects off a canyon wall and is heard shortly after it is formed
   (c) a wave refracts and reflects due to changes in the medium
   (d) two identical waves moving different directions along the same medium interfere
2. How many nodes and anti-nodes are shown in the diagram?



3. Draw a transverse wave that is reflected from a fixed end.
4. Draw a transverse wave that is reflected from a free end.
5. A wave travels along a string at a speed of $1,5\,m\cdot s^{-1}$. If the frequency of the source of the wave is 7,5 Hz, calculate:
   (a) the wavelength of the wave
   (b) the period of the wave

Appendix A

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